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**Geometry Effects
in
Tilting - Pad Journal Bearings**

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Geometry Effects in Tilting-Pad Journal Bearings

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A theoretical study has been undertaken to show the influence of bearing geometry on the steady-state and dynamic behavior of tilting-pad journal bearings. The computer model used takes into account a different viscosity on each pad, turbulence in the oil film and pad inertia.

The geometric changes considered include the pad clearance and the bearing clearance, the length/diameter ratio, the number of pads, and the orientation of the bearing with respect to the load direction. The major operating characteristics which have been examined are oil film thickness, pad temperature, power loss, and oil film stiffness and damping.

The basic form of bearing against which changes in geometry are compared has 5 centrally pivoted pads with a length/diameter ratio of 0.4. The bearing diameters considered in the theoretical study are 160 mm and 400 mm. Comparisons with experiment are also made for bearings having diameters of 160 mm and 430 mm.

INTRODUCTION

Tilting-pad journal bearings are finding increasing use in high-speed machinery particularly in applications where plain cylindrical bearings might present problems with self-induced vibration.

This paper describes the calculated effects of various geometric factors on steady-state and dynamic behavior of this kind of bearing.

Data on this behavior are presented in dimensional form so that, in examining trends, the heat balance in the bearing is taken into account. The theoretical model allows for different temperatures (and thus different viscosities) on each pad.

THEORETICAL MODEL

The theoretical performance characteristics presented here for a complete tilting-pad journal bearing are obtained by aggregating the values of those characteristics for the individual pads comprising the bearing.

NOMENCLATURE

$B_{xx}, B_{yy}, B_{xy}, B_{yx}$ = damping* (Ns/m)

c_d = diametral pad bore clearance ($2c_r$) (mm)

c'_d = diametral bearing clearance ($2c'_r$) (mm)

c_r = radial pad bore clearance ($r_p - r$) (mm)

c'_r = radial bearing clearance ($r_b - r$) (mm)

d = journal diameter (mm)

e = journal eccentricity (from bearing center) (see Fig. 1)

H = power loss (kW)

h = oil film thickness (m)

h_{min} = minimum oil film thickness (mm)

$K_{xx}, K_{yy}, K_{xy}, K_{yx}$ = stiffness* (N/m)

l = bearing length (mm)

m = preset (or preload) (i.e. $1 - c'_r/c_r$)

N = journal rotational speed (rev/min)

n = number of pads

p = oil film pressure (N/m²)

Q_{in} = inlet flow to pad (m³/s)

Q_{out} = flow from trailing edge of pad (m³/s)

Q_{side} = total flow from side of pad (m³/s)

r = journal radius (mm)

r_b = bearing radius (see Fig. 1) (mm)

r_p = pad radius (see Fig. 1) (mm)

t = time (s)

U = sliding velocity (m/s)

W = externally applied load (N)

x, y = coordinate directions

β = angle between direction of externally applied load and datum pivot line (see Fig. 1)

e' = bearing eccentricity ratio (e/c'_r)

η = dynamic viscosity

Θ_{in} = oil temperature at inlet to pad (°C)

Θ_{eff} = effective temperature in pad oil film (°C)

Θ_{max} = maximum pad temperature (°C)

ϕ = angle between direction of externally applied load and line of journal and bearing centers (see Fig. 1)

*The convention for the suffices of the stiffness and damping terms is best illustrated by an example. The term, suffixed xy is the rate of change of the x component of load with movement, either displacement or velocity, in the y direction, where the x and y directions are those defined in Fig. 2.

The pad characteristics are calculated in dimensionless terms from a finite difference solution of Reynolds equation. The form of equation used can take into account the effects of turbulence. Details of this equation, together with the boundary conditions and assumptions made in its solution, are given in Appendix 1. Dynamic coefficients are derived using small perturbation theory.

Although the characteristics for individual pads are calculated using isoviscous theory, the model for the complete bearing does allow for different temperatures on each pad, since large variations in these temperatures are observed in practice (1). An individual heat balance is performed on

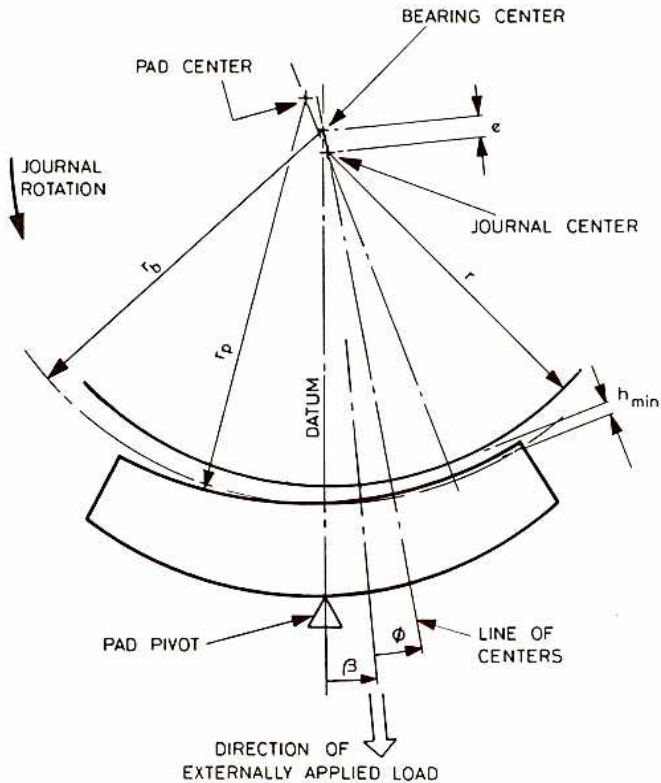


Fig. 1—Bearing notation

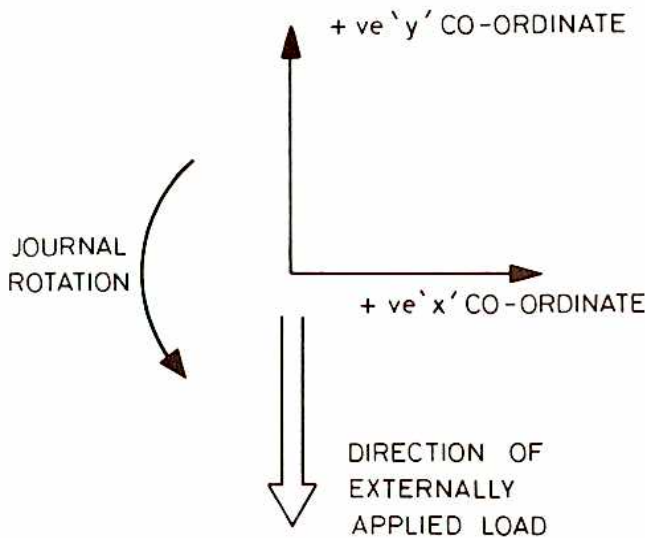


Fig. 2—Coordinate system for dynamic characteristics

each pad to determine its effective temperature and hence its effective viscosity. Although not a full thermo-hydrodynamic solution, as in (2), where variations of viscosity within the pad are taken into account, the approach used here is more realistic than assuming a single viscosity for the complete bearing (3), (4), (5). Details of the heat balance are given in Appendix 2.

The dimensionless pad characteristics, both steady-state and dynamic, used in the model have been confirmed with other theoretical results (6). By using an iso-viscous lubricant, comparisons have also been made with theoretical predictions for complete bearings (3), (4), (5). Here, the agreement is quite good, although there are some differences in absolute values due to the differing methods of solution.

Initially predictions of the model for power loss and maximum pad temperature were checked against experimental results from a 160 mm diameter bearing; geometric details of this bearing are given in Table 1. (Some of these results from unpublished experimental work carried out in the authors' company are shown in Fig. 9).

Comparisons have also been made with some published experimental data (7) for a 430 mm diameter bearing. As shown in Fig. 3, there is good agreement for power loss in the laminar regime and also when the oil film becomes turbulent. The predictions of maximum pad temperature (Fig. 4), for this size of bearing, compare quite well considering that the empirical relationship used was derived from results for a 160 mm diameter bearing.

Values of the dynamic characteristics of tilting-pad journal bearings have been determined experimentally (2), (8). However, there is insufficient published information to allow direct comparisons to be made with predictions of the theoretical model used herein.

CLEARANCES

One of the most important variables governing the performance of a hydrodynamic journal bearing is the clearance. In a plain journal bearing, the clearance and the journal eccentricity together define the film shape which controls the hydrodynamic pressure generation within the bearing. In a tilting-pad journal bearing, the situation is not so simple as there are two clearances. One, herein termed the "pad clearance," c_r , is defined as the difference in the radius of curvature of the pad profile and journal radius,

Bearing diameter	160 mm	Load on pads			
Bearing length	64 mm	oil—heavy turbine grade			
5 pads—60° arcs		Inlet temperature 50°C			
Speed (rev/min)		1000	2000	4000	6000
Bearing Clearance (mm)*		0.208	0.192	0.176	0.160
(operating)					
Preset		0.59	0.625	0.66	0.69

*Clearances used for the predictions in Figs. 9, 10, 13, and 14.

and the other, herein termed the "bearing clearance," c'_r , is the radial gap, when the journal is concentric with the bearing, between the journal and the pad at the pivot position. i.e.

$$c_r = r_p - r$$

$$c'_r = r_b - r$$

The possible displacement of the journal within the bearing is a function of the latter clearance.

Preset (or Preload)

The relationship between the two clearances is termed the preset (or preload) and is defined

$$m = 1 - \frac{c'_r}{c_r}$$

Fig. 5 shows diagrammatically how the two clearances affect the film shapes in the bearing. To produce positive preset, the pad clearance must be greater than the bearing clear-

ance. When they are equal, the preset is zero. Values of preset normally range from 0 to about 0.7.

Effect of Clearance on Pad Loading

With an eccentric journal in a bearing which has no preset, only some pads (because of film shape) generate a hydrodynamic pressure which loads the pad against its housing. With preset, however, it is possible for all the pads, including those in the "top" of the bearing, to be loaded. Pads which are "unloaded" contribute nothing to the stiffness of the bearing, but they may produce a small amount of damping (5). (However, in the theoretical predictions in this paper, no account is taken of such damping.) These unloaded

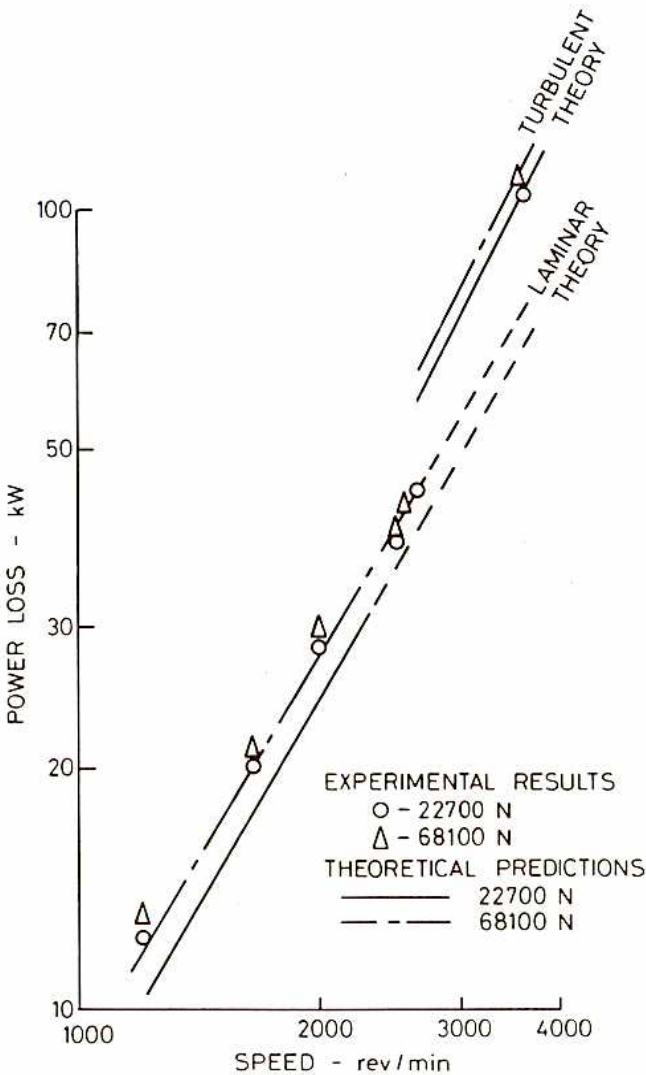


Fig. 3—Comparison of theoretical predictions and experimental results of power loss for a 430 mm diameter 5-pad bearing.

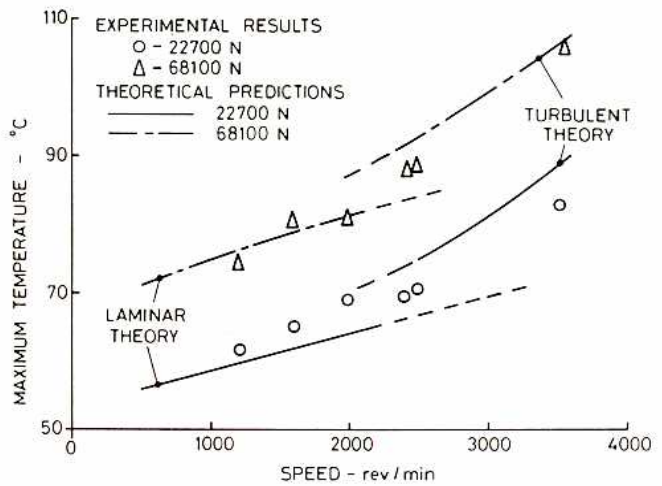


Fig. 4—Comparison of theoretical predictions and experimental results of maximum temperature for a 430 mm diameter 5-pad bearing.

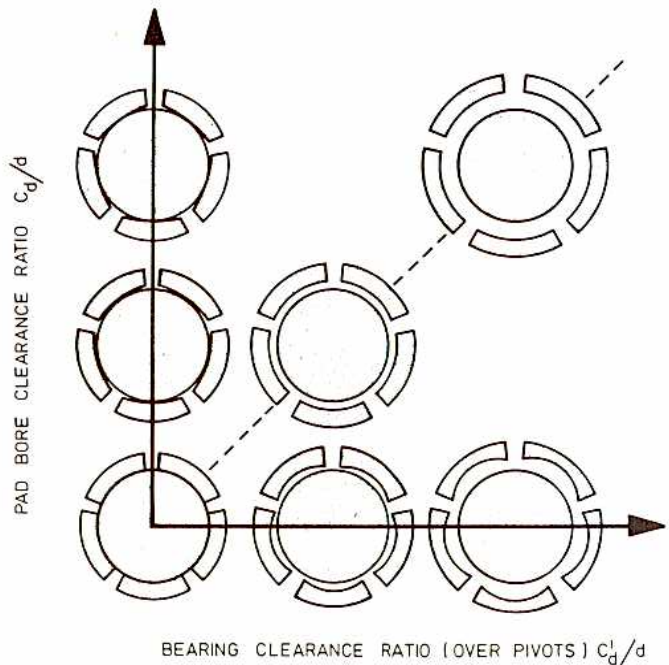


Fig. 5—Effect of clearances on film shape

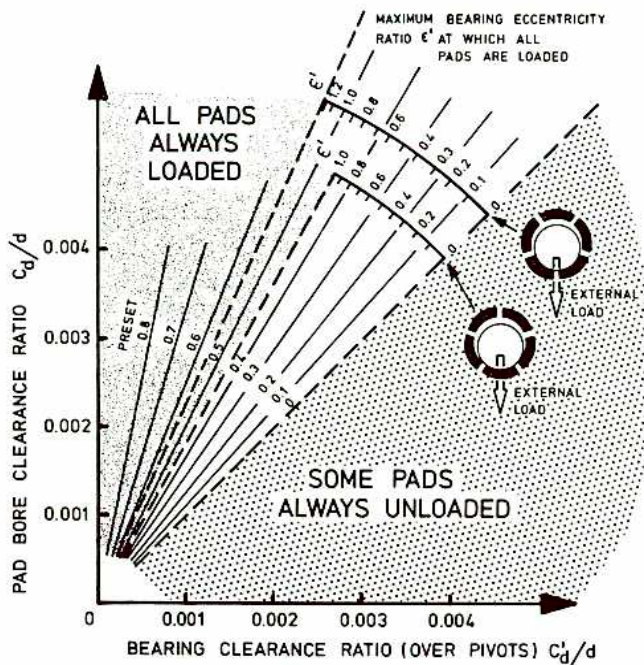


Fig. 6—Condition for all pads of a 5-pad bearing to be loaded

pads, and even lightly loaded ones, may be unstable and lead to pad “flutter.” Therefore, it is a desirable feature to have all the pads in a bearing loaded.

The condition for all pads to be loaded is purely a function of the two clearances in the bearing and the journal position. Data are given in Fig. 6 for determining whether this condition is satisfied. For any value of preset, there is a value of eccentricity below which all the pads are loaded; this value is dependent on load direction.

It is of interest to note that, when loaded between pads, the maximum journal displacement can exceed the bearing clearance; in effect, the journal can “fall between” two pads. A 5-pad bearing can attain an eccentricity ratio of nearly 1.24.

Operating Clearance

One difficulty that does arise with a tilting-pad journal bearing is knowing the actual (or effective) value of the two clearances under real operating conditions. There are several reasons for this, not least of which is the influence on clearance of the manufacturing tolerances on the various bearing components. Also, the bearing pad, being unconstrained, will distort, because of load and because of thermal gradients within the pad, and alter the effective pad clearance. The bearing clearance will also be affected by differential expansion of the journal and the bearing, especially under cold start-up conditions where heating of the journal and pad is more rapid than that of the housing. The loss of clearance may be so severe that the bearing seizes (9).

TRENDS IN STEADY LOAD PERFORMANCE

The effects of geometry upon four steady-state characteristics are considered, namely: power loss, maximum pad temperature, journal eccentricity, and minimum film thickness.

Power loss is an important factor in bearing design, particularly at high speed because of energy consumption and cooling requirements.

Pad maximum temperature may be a limiting factor on performance (10) depending on the properties of the bearing material.

The journal eccentricity is not of direct importance to the operation of the bearing itself, but may be so for external reasons, e.g. the possible rubbing of the shaft on glands or seals.

The minimum film thickness may be a limiting factor especially at low speed. An allowable minimum value is a function of the roughnesses of the journal and bearing surfaces.

(The reason for considering both eccentricity and minimum film thickness is that, in a tilting-pad journal bearing, they do not have a direct relationship as in a plain cylindrical bearing.)

Clearances

The effects of the two clearances in the bearing on its steady-state characteristics are shown in Figs. 7 and 8. One figure shows results for a 160 mm diameter bearing and the other for a 400 mm bearing.

The main controlling influence on power loss is the bearing clearance which, when increased, leads to a reduction in power loss. Increasing the pad clearance can produce a slight increase in power loss under high load/low speed conditions.

An increase in bearing clearance also tends to reduce maximum temperature, as does an increase in pad clearance.

For a given operating condition, the journal eccentricity is almost directly proportional to the bearing clearance whereas it is almost independent of pad clearance.

Minimum film thickness is relatively insensitive to changes in either clearance, the effects being a function of the actual values of load and speed.

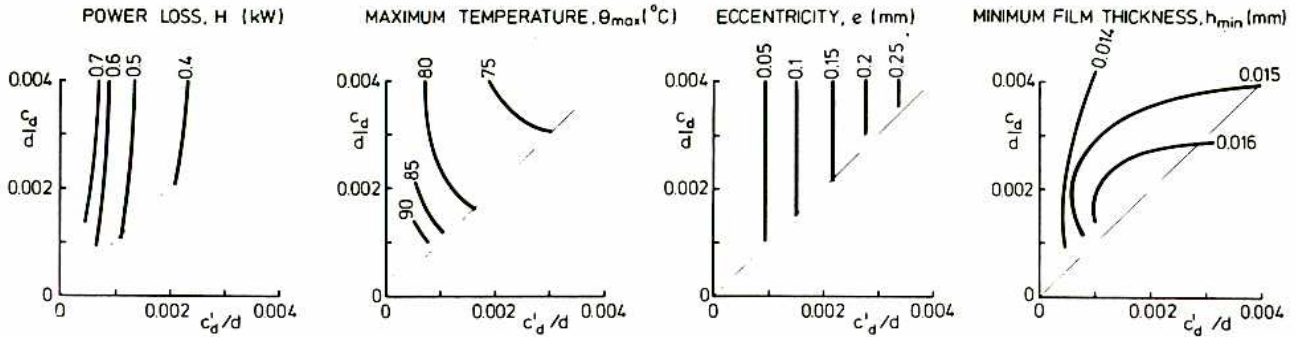
Effect of Load and Speed

The main graphs in Figs. 9 and 10 show characteristic trends in steady-state performance for a 160 mm diameter 5-pad bearing. Due to thermal effects, the actual values of clearances in the bearing tend to change with operating conditions and so the values of clearances used in these predictions are based on measured values from a test bearing operating under the same load/speed conditions (see Table 1).

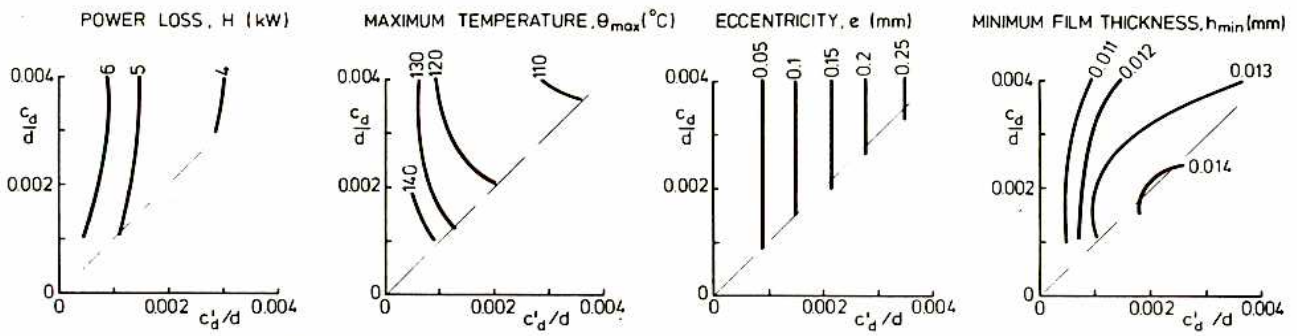
From Fig. 9, it can be seen that the main controlling influence on power loss is speed, whereas load has little effect. Increases in both load and speed raise the maximum pad temperature.

Increasing the load tends to increase eccentricity (Fig. 10), whereas raising the speed tends to reduce it. Generally, these changes have the opposite effect on minimum film thickness except under low load/high speed conditions.

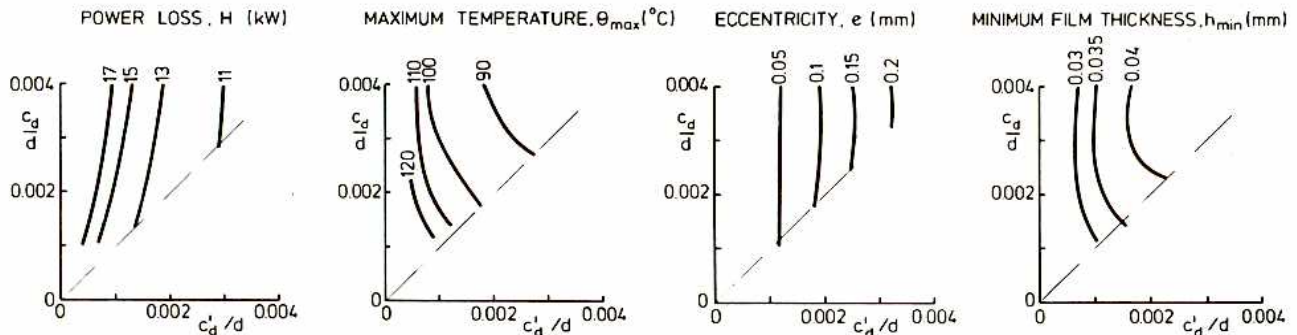
Around each of the main graphs (Figs. 9 and 10) is a set of smaller graphs showing the same characteristics, but with a change in one of the bearing parameters. The effect of



(a) - LOAD 10 240 N - SPEED 1000 rev/min



(b) - LOAD 25 600 N - SPEED 4000 rev/min



(c) - LOAD 10 240 N - SPEED 8000 rev/min

5 PADS
 DIAMETER 160 mm
 L/d RATIO 0.4
 LOAD ONTO PAD
 MEDIUM TURBINE OIL
 INLET TEMPERATURE 50°C

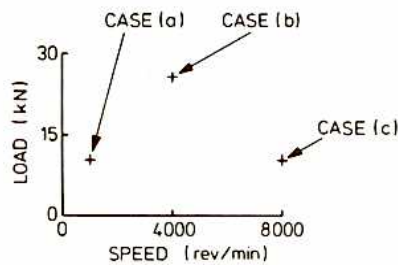
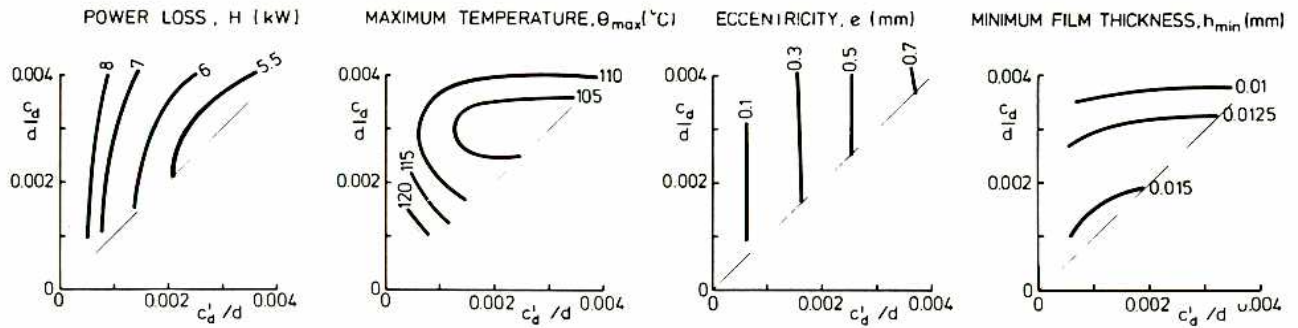
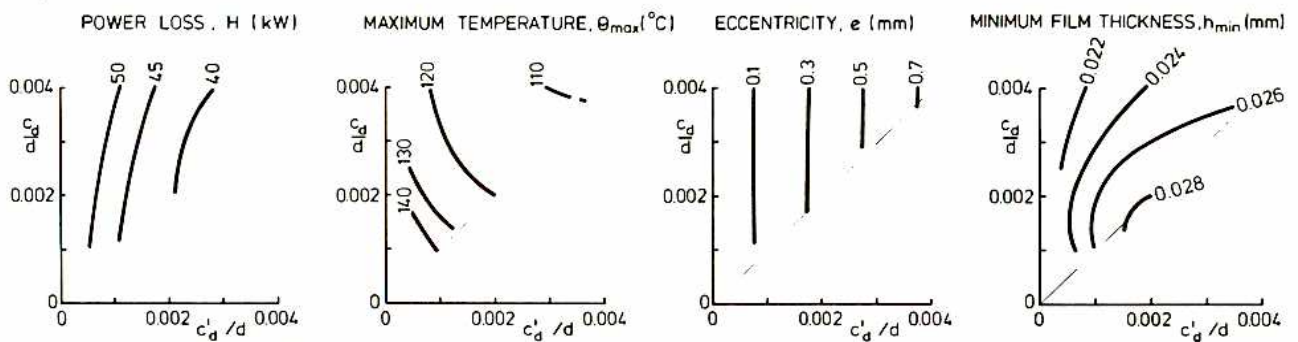


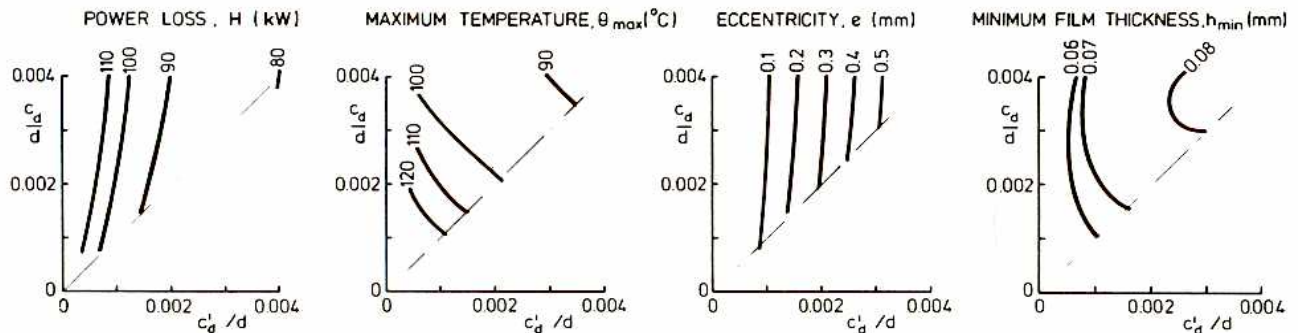
Fig. 7—Effect of clearances on steady-state characteristics of a 160 mm diameter bearing.



(a) - LOAD 160 000 N - SPEED 800 rev/min



(b) - LOAD 160 000 N - SPEED 2400 rev/min (Turbulent)



(c) - LOAD 64 000 N - SPEED 3600 rev/min (Turbulent)

5 PADS
 DIAMETER 400 mm
 L/d RATIO 0.4
 LOAD ONTO PAD
 MEDIUM TURBINE OIL
 INLET TEMPERATURE 50°C

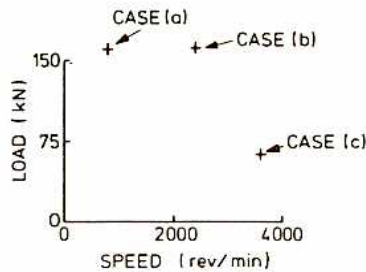


Fig. 8—Effect of clearances on steady-state characteristics of a 400 mm diameter bearing.

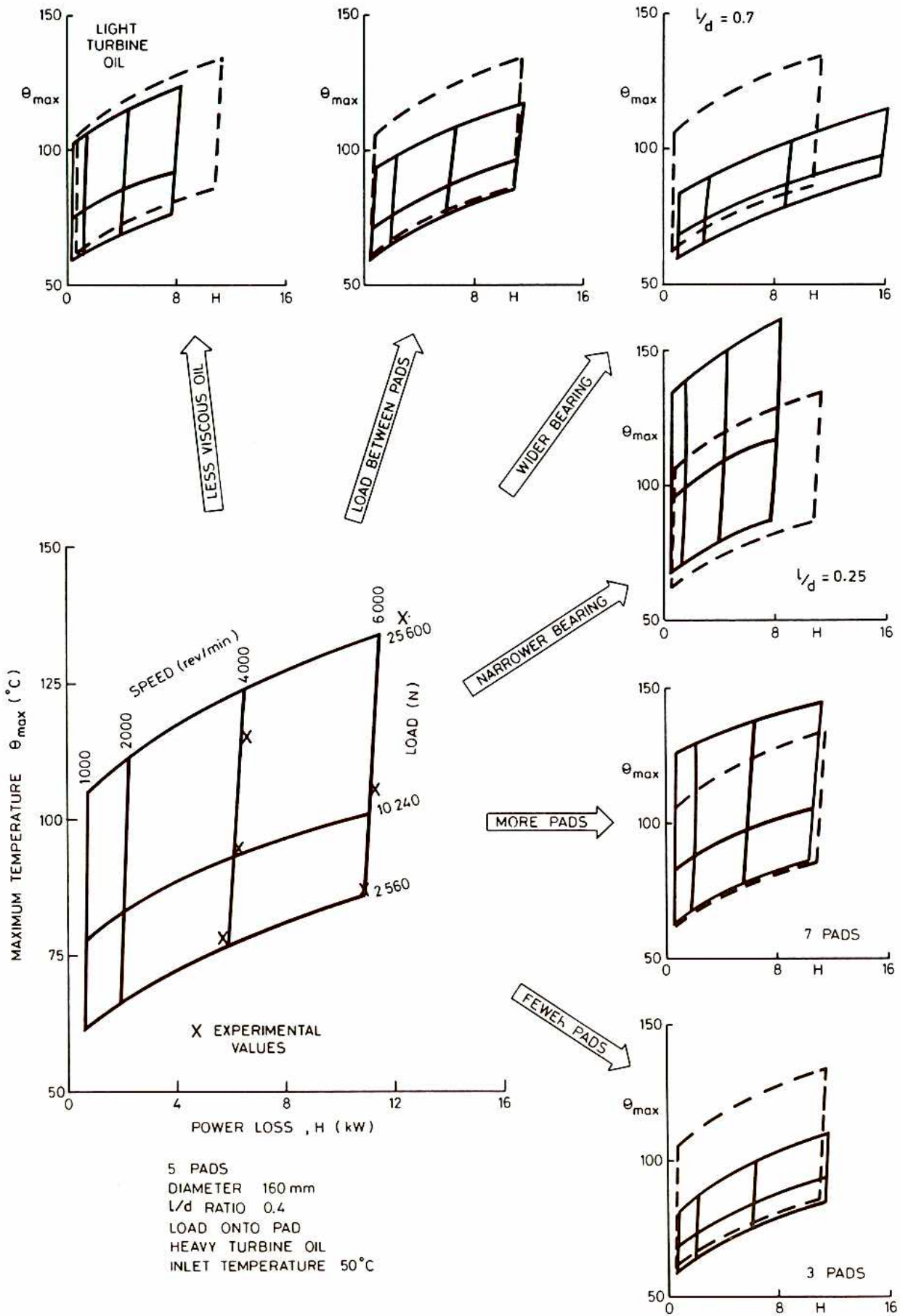


Fig. 9—Effect of various bearing parameters on power loss and maximum temperature.

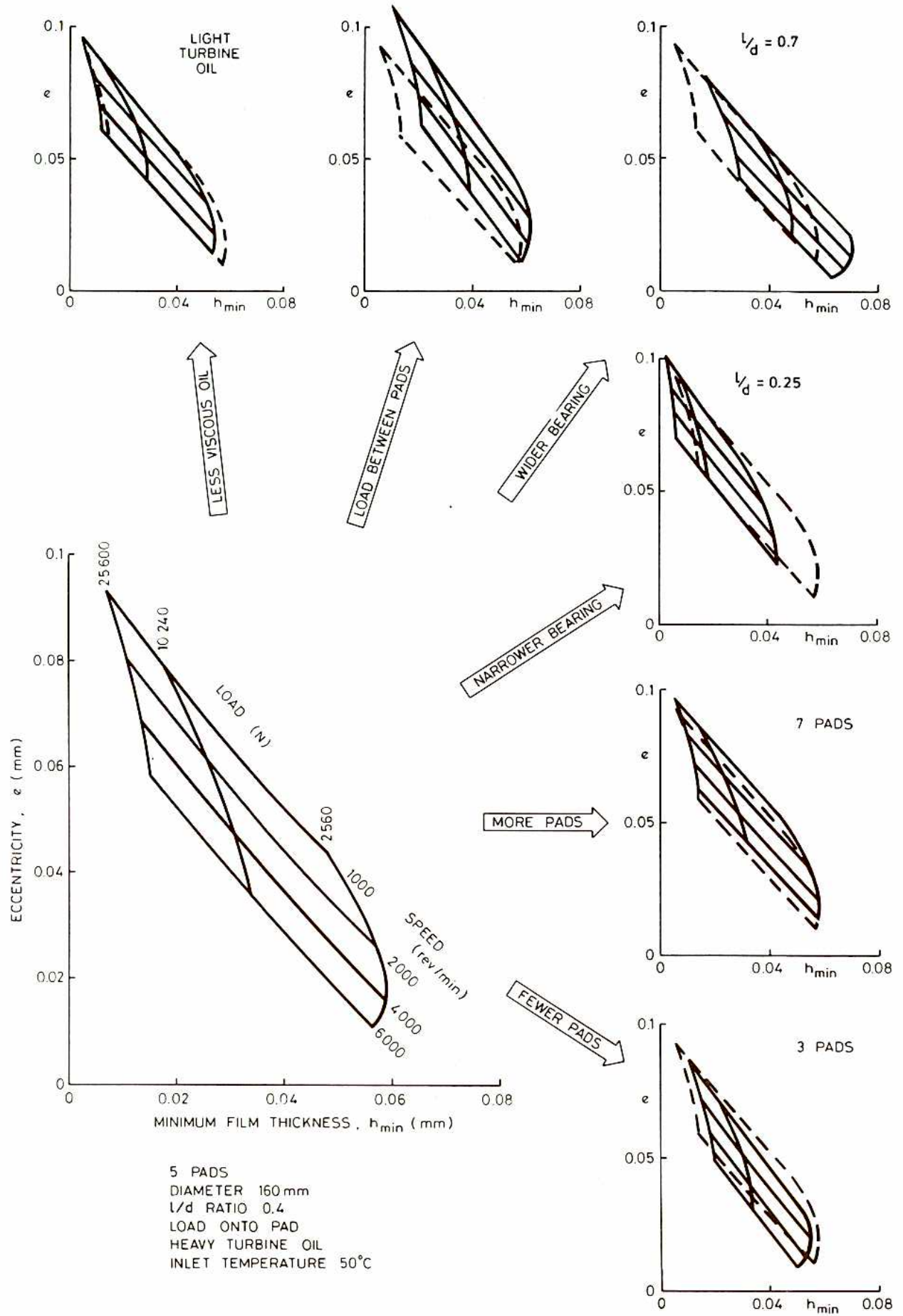


Fig. 10—Effect of various bearing parameters on minimum film thickness and eccentricity.

changing the various bearing parameters can be seen by comparing the position of the load/speed grid on each of these small graphs with its position on the main graph (also shown dotted on each small graph).

Load Direction

Generally, a tilting-pad journal bearing is loaded symmetrically, i.e. either onto a pad pivot or between two pivots, the former being the more common situation.

It can be seen from Fig. 9 that changing the direction from "on pad" to "between pads" tends to reduce the maximum temperature in the bearing, especially at high load, but to have little effect on power loss. This change in load direction increases minimum film thickness (Fig. 10) substantially so under high load/low speed conditions, but at the expense of increased journal eccentricity.

The bearing may be loaded in any direction, not just in these two special directions. Figure 11 shows bearing characteristics where the load direction has been varied gradually from a "between pad" position to the next "between pad" position. This graph endorses the results in Figs. 9 and 10 and also shows that there is a gradual change in steady-state characteristics as the load direction is changed between symmetrical positions.

Bearing Length

There is an increase in power loss with increasing length (Fig. 9), mainly as the result of the change in bearing surface area. The maximum pad temperature is reduced.

A longer bearing leads to reduced eccentricity (Fig. 10); there is an increase in minimum film thickness especially under high load conditions.

Number of Pads

Changing the number of pads (while maintaining approximately the same total pad area) has most effect on maximum pad temperature (Fig. 9), which has a lower value with fewer pads. Power loss, however, is very little affected.

The number of pads does not significantly affect minimum film thickness (Fig. 10) except for some increase under high load conditions with three pads. Eccentricity is slightly reduced with fewer pads, but this trend would be reversed if the bearing were loaded between pads as the journal would then "fall" further between more widely spaced pads.

Lubricant

Although the lubricant is not a geometric factor, the effects of changing its grade have been included here in an attempt to "complete the picture."

Figure 9 shows that using a less viscous oil results in lower power loss and lower maximum temperature. Eccentricity and minimum film thickness (Fig. 10) are not affected greatly except at high speed where there is a slight decrease in minimum film thickness.

(Specifications of oils used in this paper are given in Table 2.)

TRENDS IN DYNAMIC CHARACTERISTICS

The rationale for using a tilting-pad journal bearing is often its superior capability of suppressing oil film whirl. A measure of this capability, for a simple two-bearing system, is the critical mass which can be calculated from the stiffness and damping coefficients for the bearing (11). For tilting-pad journal bearings, this critical mass is invariably found to be infinite, indicating that the bearing is inherently stable. However, it is of interest to study the effects of geometry on the values of the dynamic coefficients, as these are required for a more comprehensive dynamic analysis of practical rotor systems.

Clearance

The influence of bearing clearance and pad clearance on dynamic characteristics is shown in Fig. 12, for a 160 mm and a 400 mm diameter 5-pad bearing. High speed cases were selected as they represent the situations in which dynamic characteristics are of most interest.

When considering the dynamic characteristics, it is the direct terms, those suffixed $_{xx}$ and $_{yy}$, that are of most interest since the cross-coupled terms, those suffixed $_{xy}$ and $_{yx}$, are typically about 3 orders of magnitude smaller. If the pad inertia is ignored, then the cross-coupled terms are found to be even smaller.

For both bearings it can be seen, from Fig. 12, that the K_{xx} stiffness term is very strongly dependent on the bearing clearance ratio, which, if changed from 0.001 to 0.004, causes a reduction of K_{xx} by two orders of magnitude. An increase in pad clearance tends to increase K_{xx} . The K_{yy} stiffness term (i.e. in the direction of the load,) is also very dependant on the bearing clearance, but not to the same degree as K_{xx} . The pad clearance has little effect on the K_{yy} term at small values of bearing clearance ratio.

The predominant influence on the direct damping terms, B_{xx} and B_{yy} , is the bearing clearance. Pad clearance has little effect on B_{xx} but can significantly affect B_{yy} .

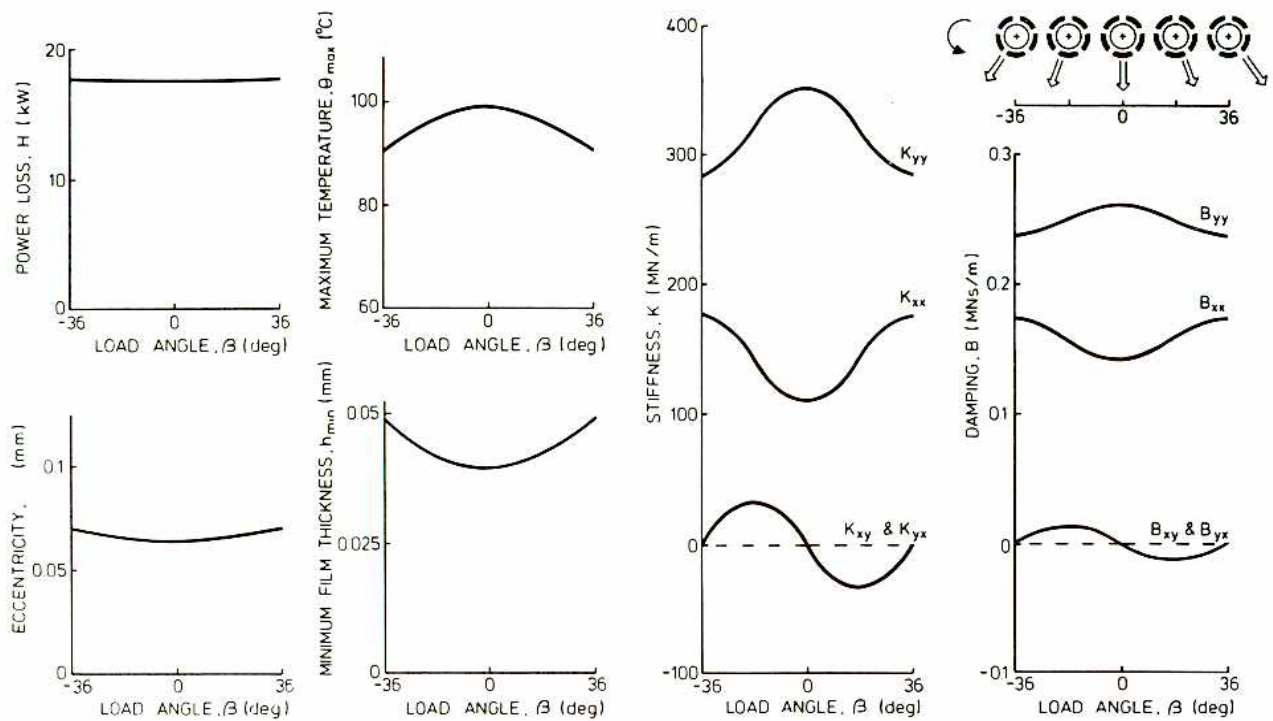
Since the cross-coupled terms are so small compared with the direct terms, it is sufficient to comment that the bearing clearance has more effect than pad clearance.

Effects of Load and Speed

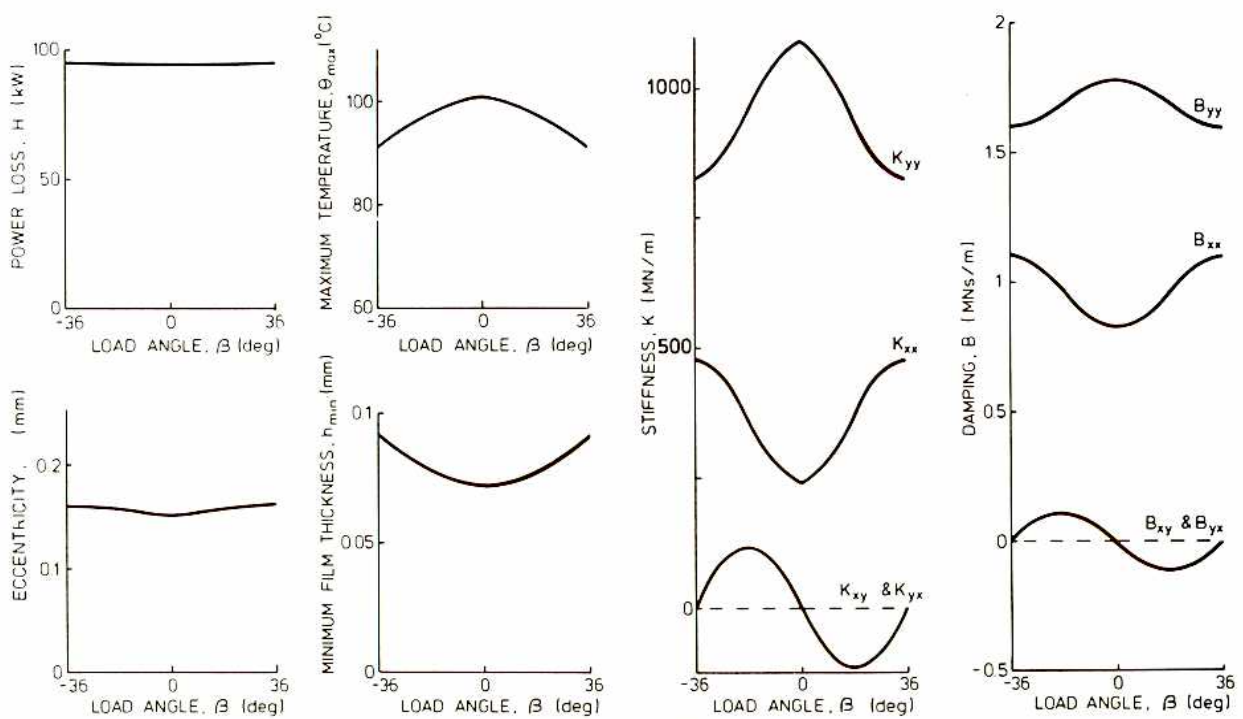
Figures 13 and 14 show the effects of various bearing parameters on the dynamic coefficients. Some general observations can be made on the effects of load and speed on stiffness and damping. For stiffness, an increase in load produces large increases in K_{yy} , particularly at low speed,

TABLE 2—VISCOSITY TEMPERATURE DATA FOR LUBRICANTS USED

Oil Grade	Dynamic Viscosity mPa.s (centipoise)		Used in Figures Numbers
	at 40°C	at 100°C	
Light Turbine	27.5	4.4	9, 10, 13, 14
Medium Turbine	40	5.7	7, 8, 11, 12
Heavy Turbine	58	7.2	9, 10, 13, 14



(a) - DIAMETER 150 mm - LOAD 10 240 N - SPEED 8000 rev/min



(b) - DIAMETER 400 mm - LOAD 64 000 N - SPEED 3600 rev/min (Turbulent)

Fig. 11—Effect of varying load direction on bearing characteristics

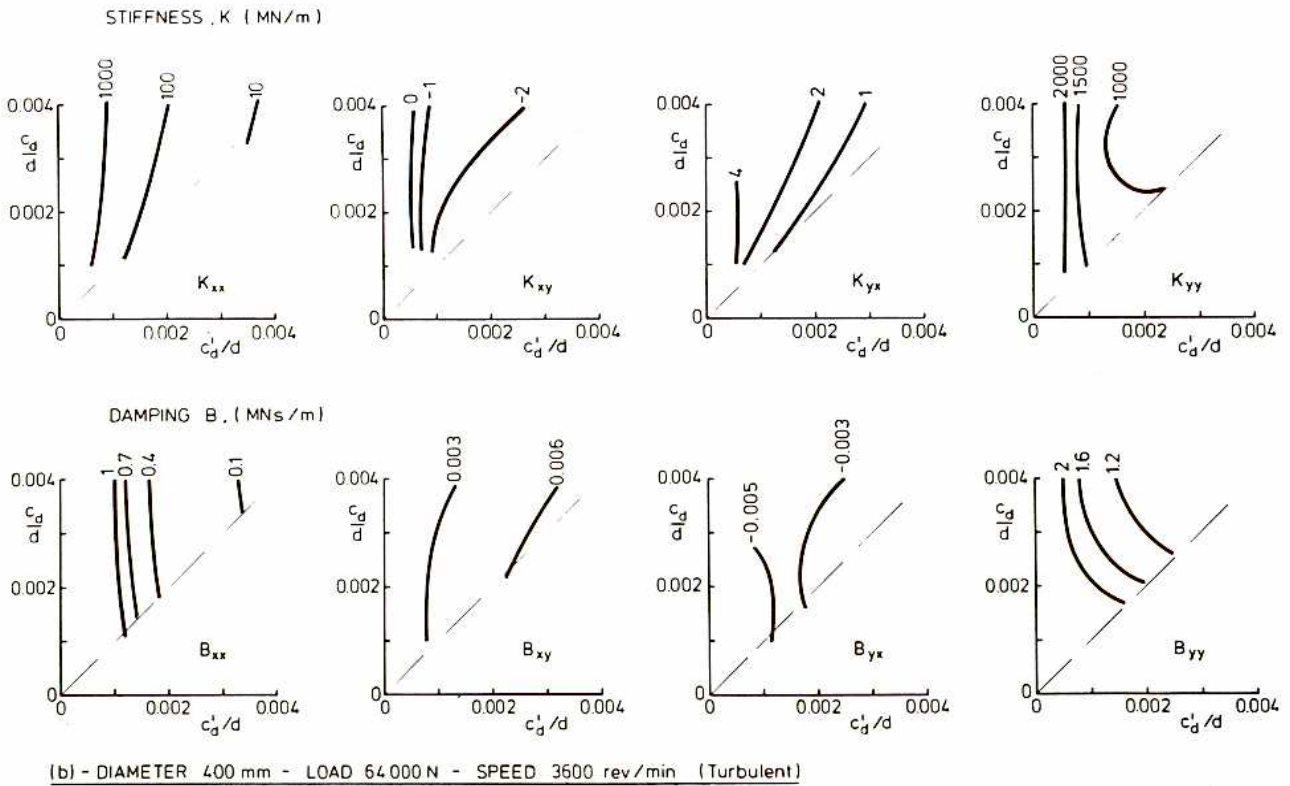
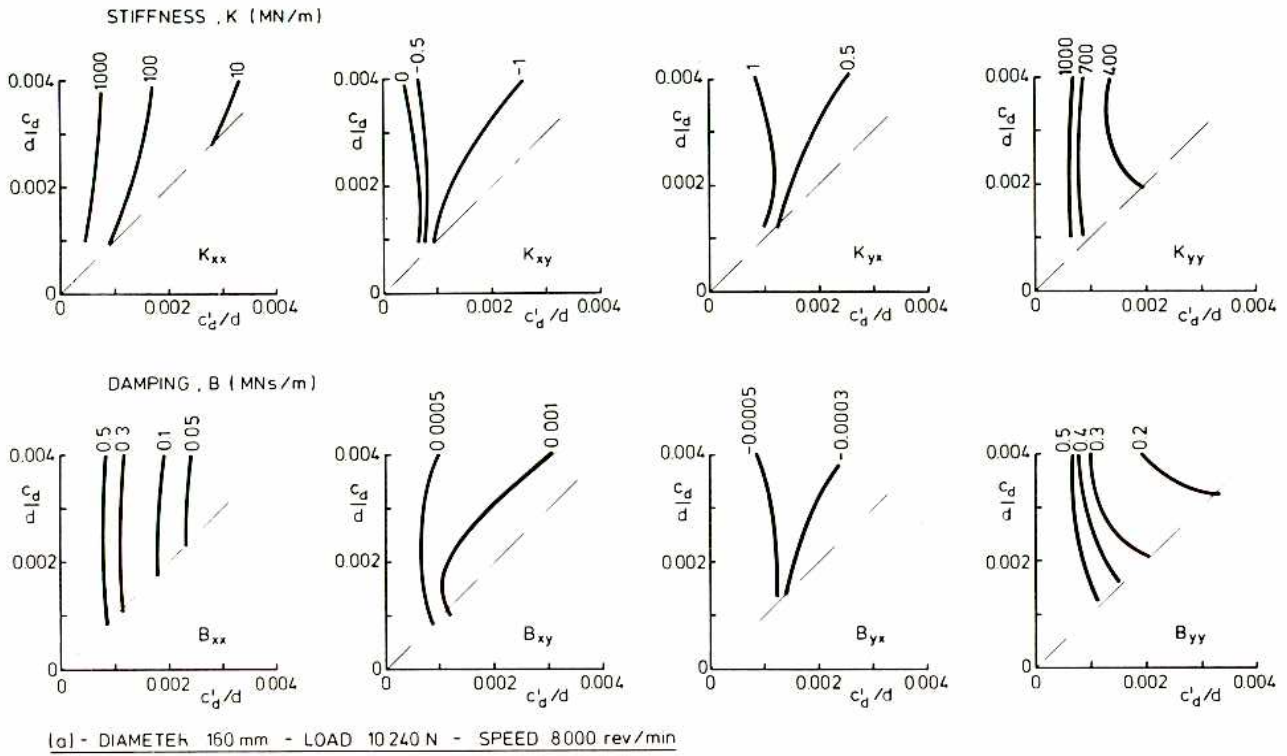


Fig. 12—Effect of clearances on dynamic characteristics

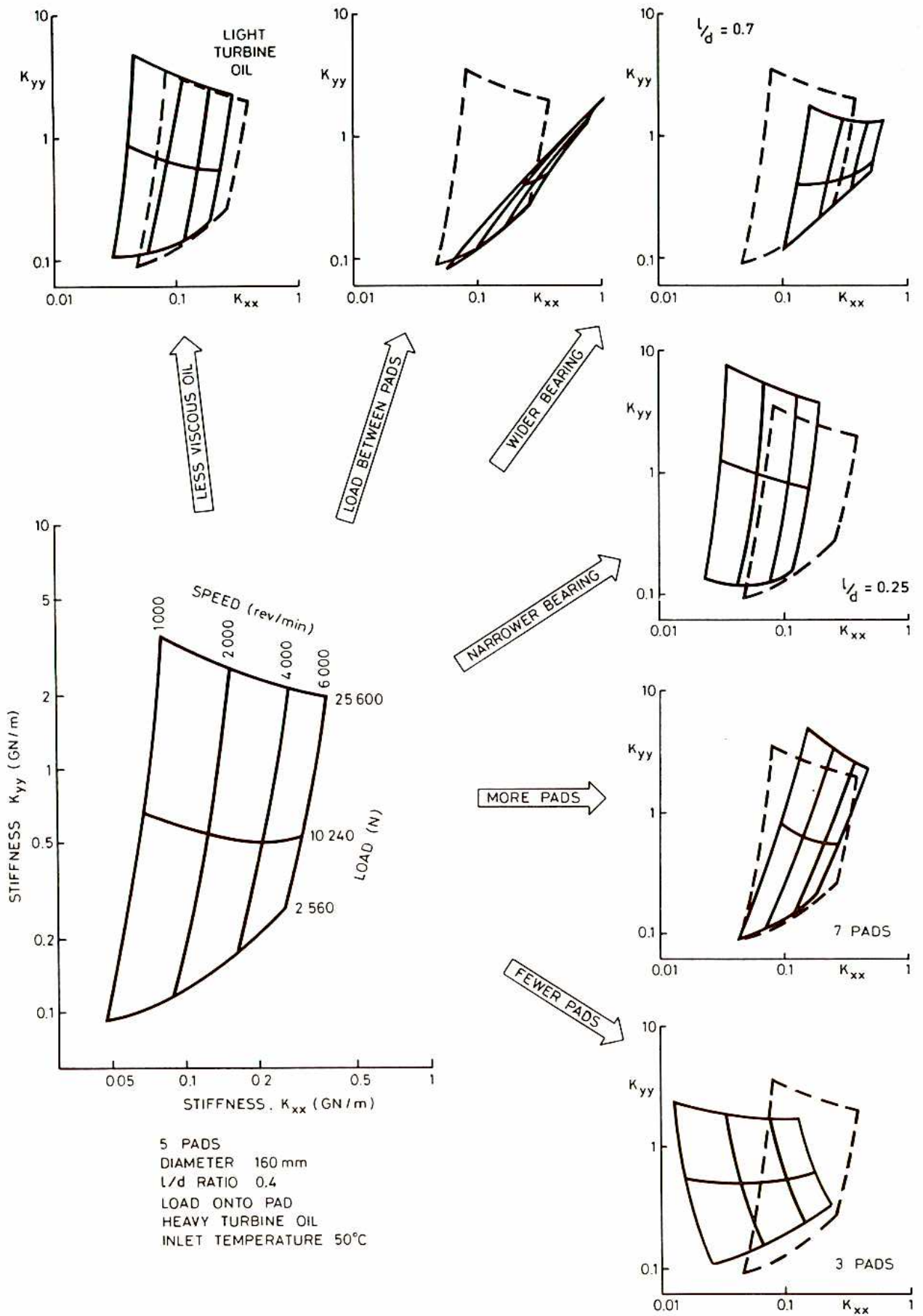


Fig. 13—Effect of various bearing parameters on stiffness

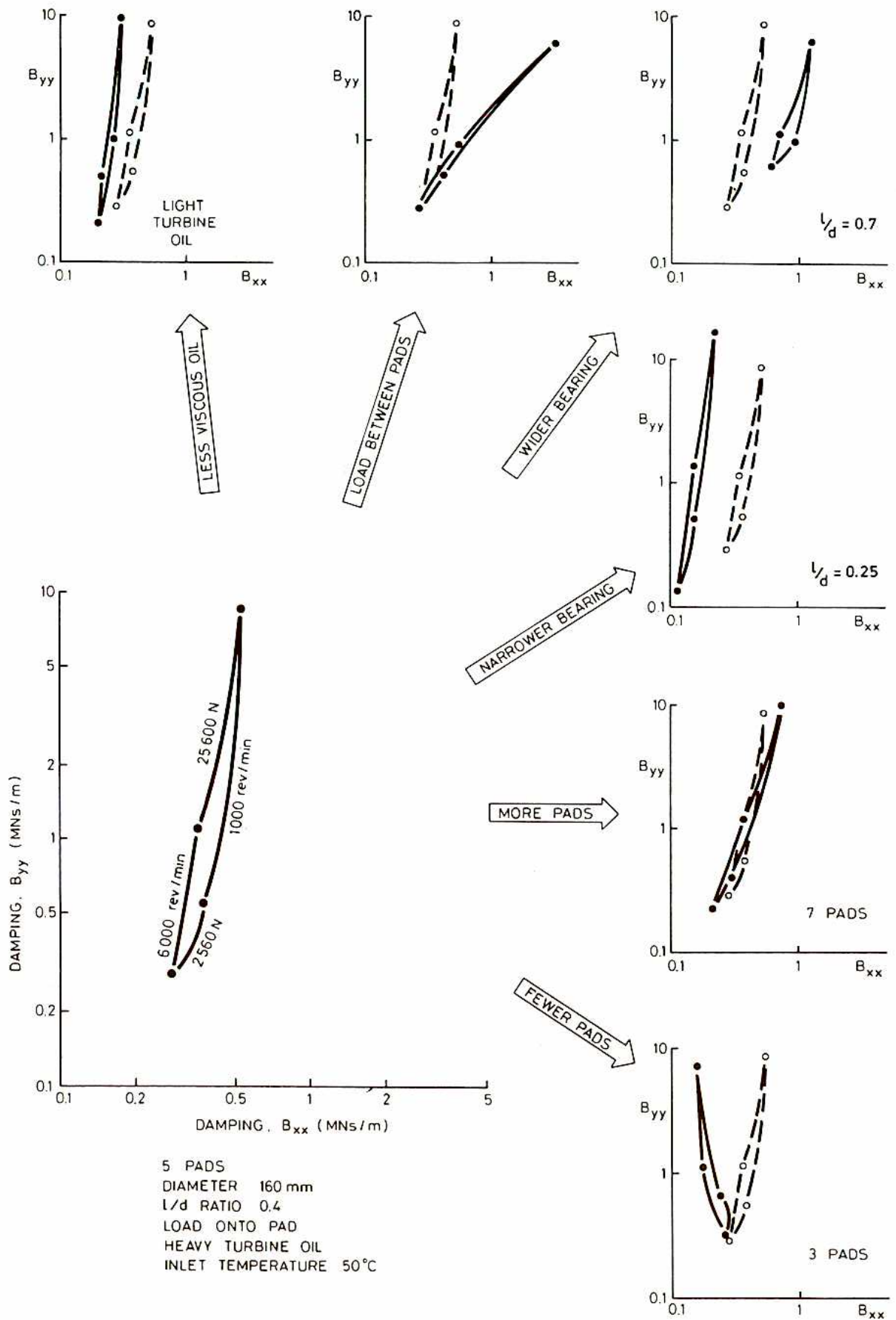


Fig. 14—Effect of various bearing parameters on damping

whereas there is only a slight effect on K_{xx} . The main effect of increased speed is to produce an increase in K_{xx} . K_{yy} increases with speed at low load but, at higher loads, this trend reverses.

For damping, both load and speed mainly affect B_{yy} , which increases with load and decreases with speed. Similar trends apply to B_{xx} , but to a much lesser degree.

Load Direction

Referring to Fig. 13, simply changing from one symmetrical loading direction, i.e. onto a pad, to the other symmetrical position, i.e. between pads, has little effect on either stiffness term at low load. At high load, there is a very marked increase in K_{xx} , especially at low speed. There is also a reduction in K_{yy} at high load.

The effects on the damping terms (Fig. 14) are very similar to those on the stiffness terms.

It can be seen from both Figs. 13 and 14 that the overall effect of changing from loading onto a pad to between pads is to bring the values of the direct dynamic terms closer together. This is borne out by the results in Figs. 11a and 11b, which show the effects of loading in any direction for a single load/speed condition.

In Fig. 11, the cross-coupled stiffness and damping terms are also shown. For the symmetrical loading directions, the values of these are insignificant compared with the direct terms. However, when the pads are not symmetrical about the load direction, the cross-coupled terms attain quite large values, the signs of which depend on whether the load is before or after the pivot, in the direction of rotation. Despite this significant cross-coupling, the bearing is still inherently stable [when using the Routh criterion (11)]. It has been found that these large values of cross-coupled terms are due to the asymmetrical loading and do not arise as a result of including the effects of pad inertia.

Bearing Length

The general effect on stiffness of using a narrower bearing (Fig. 13) is to reduce the value of K_{xx} . There is an increase in K_{yy} at high load, but at low load the value may increase or decrease depending on the speed. It is of interest to note that for the wider bearing ($l/d = 0.7$) K_{xx} tends to become equal to K_{yy} under low load conditions.

Using a narrower bearing reduces the value of the damping term B_{xx} (Fig. 14). The effect on the value of B_{yy} is to increase it at high load/low speed conditions and decrease it at low load/high speed.

Number of Pads

Figure 13 shows that increasing the number of pads in the bearing from five to seven tends to increase the values of both direct stiffness terms at high load, but to decrease the value of K_{xx} at high speed/low load conditions. Reducing the number of pads from five to three alters one of the trends in stiffness in that K_{xx} then tends to decrease with increasing load; this results in the values of K_{xx} being very much reduced at high load. There is little change in the value of K_{yy} , for either an increase or decrease in the number of pads.

Using seven pads instead of five shows little effect on the damping terms (Fig. 14). Reducing the number of pads from five to three, however, results in a much lower value of B_{xx} at high load/low speed conditions although little change at low load/high speed. Change in the number of pads has little effect on B_{yy} .

The explanation for the marked effect on the $_{xx}$ terms in changing from five to three pads is that, in the latter case, the only pads contributing to those terms are in the "top" of the bearing. With increasing load, the journal moves away from the top pads, thereby reducing their contribution to the $_{xx}$ terms; so, in a three-pad bearing, the values of these terms are reduced. However, with five or more pads, there are some pads which, being in the "bottom" of the bearing, contribute more to the $_{xx}$ terms with increasing load and so these terms have higher values.

Lubricant

Using a less viscous oil has little effect on K_{yy} and B_{yy} , but the values of K_{xx} and B_{xx} are reduced particularly under low speed conditions.

CONCLUSIONS

Bearing clearance is more significant than pad clearance to both steady-state and dynamic characteristics. Pad clearance has some effect on maximum pad temperature.

Loading between pads reduces maximum pad temperature and increases minimum film thickness. It also tends to reduce the $_{yy}$ dynamic terms and increase the $_{xx}$ terms. A nonsymmetrical direction of loading can produce significant values of the cross-coupled dynamic terms.

Reducing the bearing length results in lower power loss, but increases maximum pad temperature and reduces minimum film thickness. The effect on the dynamic characteristics is to reduce the $_{xx}$ terms.

The most significant effect of number of pads is on maximum pad temperature, which is reduced with fewer pads, especially at high load. A 3-pad bearing has significantly lower values of $_{xx}$ dynamic terms, at high load, than either a 5-pad or 7-pad bearing.

The main effects of using a less viscous lubricant are to reduce power loss and maximum pad temperature.

ACKNOWLEDGMENTS

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APPENDIX 1

For determining the basic pad characteristics used in the theoretical model for a tilting-pad journal bearing, the following form of Reynolds equation was used

$$\frac{\partial}{\partial x} \left(G_x \frac{h^3 \partial p}{\eta \partial x} \right) + \frac{\partial}{\partial y} \left(G_y \frac{h^3 \partial p}{\eta \partial y} \right) = \frac{U}{2} \frac{\partial h}{\partial x} + \frac{\partial h}{\partial t}$$

This form of equation was derived by Ng and Pan (12) using a linearized turbulence theory. The coefficients G_x and G_y are functions of Reynolds number under turbulent conditions. For laminar conditions their value is 1/12, thus yielding the more familiar form of Reynolds equation.

This equation is solved numerically by finite difference techniques. The boundary conditions applied are:

1. that there is always a full film at the leading edge of the pad
2. that the pressure, p , is zero on the edges of the pad
3. that at film rupture $p = \frac{\partial p}{\partial x} = 0$.

In a ruptured region, the film is assumed to break up into streamers, thus reducing the wetted area. In calculating the power loss, the friction at an axial section, within such a region, is reduced by the ratio of the film thickness at the upstream rupture boundary to the film thickness at that section. It is also assumed that, since there is no pressure generated in the ruptured regions, they do not contribute to the stiffness and damping of the pad.

For any given journal position, the tilt of a loaded pad is such that the net pad load acts through the pad pivot. However, for an unloaded pad, i.e. a pad which has a crescent-shaped lubricant film when its pivot is against the housing, it is assumed that the lubricant film is completely ruptured, hence no pressure is generated and, therefore, the pad makes no contribution to the stiffness and damping of the complete bearing.

For purposes of calculating power loss on an unloaded pad, it is assumed that the pad does not tilt relative to the journal.

APPENDIX 2

Having determined the performance characteristics of a pad in dimensionless terms, the value of the effective viscosity is needed in order to calculate the actual characteristics. This viscosity is dependant on the effective pad temperature Θ_{eff} which is determined by performing a heat balance.

The temperature rise in the pad is a function of the shear losses and the lubricant flow through the pad, i.e.

$$\Theta_{eff} - \Theta_{in} = \frac{k H_{pad}}{C \rho Q_{cooling}} \quad [1]$$

where k is the proportion of the total heat generated that is carried away by the lubricant (assumed to be 0.85) and C and ρ are the heat capacity and density of the lubricant. Referring to Fig. 15, it can be seen that all the outlet flow over the trailing edge of the pad, Q_{out} , cools the bearing, whereas only part of the side flow effectively does so, since that side flow leaving the pad towards its leading edge carries away very little of the total heat generated. Therefore, the assumption is made that the effective cooling flow is:

$$Q_{cooling} = Q_{out} + \frac{1}{2} Q_{side} \quad [2]$$

The heat balance Eq. [1] has to be solved iteratively, as both the power loss, H_{pad} , and the flows are a function of the effective viscosity and hence Θ_{eff} . This heat balance must be performed for each pad in the bearing.

The following approximation is used to determine the maximum temperature on each pad.

$$\Theta_{max} = \Theta_{eff} + \frac{2}{3} (\Theta_{eff} - \Theta_{in}) \quad [3]$$

The maximum temperature in the bearing is the highest of the individual pad values.

In determining the appropriate inlet temperature, Θ_{in} ,

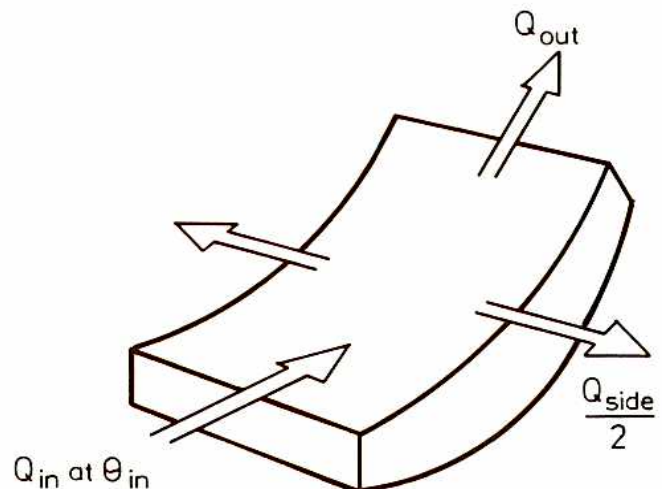


Fig. 15—Flows and temperatures for pad heat balance

for each pad, account is taken of "hot oil" carried over from the previous pad. This is done by assuming that a proportion of outlet flow, Q_{out} , from the previous pad, at its effective temperature Θ_{eff} , is available to feed the inlet to the next pad. If this flow is insufficient to fill the pad inlet, then the rest of the pad inlet flow is "made up" with lubricant at a

temperature somewhere between the inlet and outlet temperatures to the bearing housing. (It is assumed in the examples in the paper that 85 percent of the "hot oil" from the trailing edge of one pad is available as inlet to the next and that the "make-up" lubricant temperature is half way between housing inlet and outlet temperature.)

DISCUSSION

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The discussor would like to congratulate the authors for their excellent contribution to tilt-pad bearing design. The results concerning the effect of various bearing parameters on tilt-pad stiffness and damping are particularly useful.

For high-speed, high-performance turbomachinery, bearing stiffness is an important consideration. For example, tilt-pad bearings that are many times stiffer than the machine's fundamental shaft stiffness may not permit enough bearing damping to be transmitted into effective damping at the rotor center. This would result in a rotor-bearing system that is very sensitive to synchronous and subsynchronous excitation. Tilt-pad bearings should be designed to match the shaft stiffness. The authors have provided a valuable design tool with their parametric study.

Have the authors compared their results with other experimental or theoretical results available in the literature? If so, these comparisons would be very valuable and should be included in the paper. Another curve that would be very helpful is a dimensional plot of stiffness and damping coefficients that compares the constant viscosity case to the variable pad viscosity case used by the authors. The effect of variable viscosity on the dynamic performance of tilt-pad bearings is extremely important, particularly when the bearing is operating at low speeds and/or high loads.

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AUTHORS' CLOSURE

The authors are grateful to Dr. Nicholas for his comments. In particular, the advice on "matching" bearing and shaft stiffness is especially welcome from one who has made recent advances in this field (B1).

In answer to the specific questions, the authors have compared their results with others. In their paper, the authors stated that there was insufficient data to compare the experimental work of Yamauchi and Someya (8). However, more data have since been made available to us by those authors and the predicted dynamic characteristics are com-

pared with the experimental results of Yamauchi and Someya in Fig. B1. Details of the bearing geometry are:

- + tilting pads—centrally pivoted
- pad arc angles 55° —loaded between pads
- journal diameter 20 mm
- bearing length 10 mm
- radial pad bore clearance 0.1 mm
- radial bearing clearance 0.057 mm
- preset, $m = 0.43$

A spindle oil with a viscosity of 15.8 centipoises at 30°C and 2.56 centipoises at 100°C was used. The oil inlet temperature was 26°C . The direct stiffness and damping shown by the curves in Fig. B1 were computed, by the authors, using a nonglobal heat balance procedure (as adopted in their paper). The general trends in the theory for this case show a good resemblance to those indicated by experiment, thus creating more confidence in the future use of the mathematical model as a design tool.

The authors have also compared other theoretical results in the literature with their predicted dynamic performance characteristics and these are summarized in Fig. B2. Two

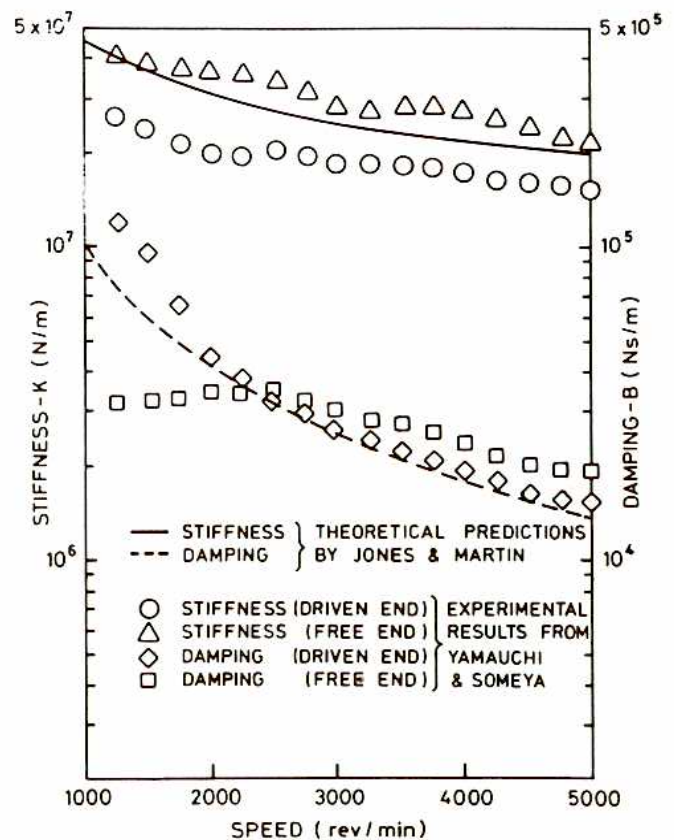


Fig. B1—Comparison of theoretical predictions and experimental measurements of dynamic characteristics for a 20-mm diameter 4-pad bearing.

sources of data are studied; one based on the work of the discussor and his coauthors, Gunter and Allaire (5) and the other based on the work of Shapiro and Colsher in (B2) and in private communication to us. The results* relate to five pad bearings with l/d ratios of 0.5 and 1.0 and presets (m) of 0.5 and zero, respectively.

When studying the various theoretical predictions for presentation in Fig. B2, it became apparent that the relationship between bearing eccentricity ratio ϵ and Sommerfeld Number S were almost identical from all sources considered. This was useful since the stiffness and damping coefficients could then be compared on a common framework of either ϵ or S as shown.

For the $l/d = 0.5, m = 0.5$ case, there was very little difference between the dynamic coefficients predicted by the author (full line), Nicholas (circles) and Shapiro (crosses). For the $l/d = 1, m = 0$ case, again there was very little difference between the various predictions for the stiffness coefficient, but a noticeable difference between the authors' damping coefficient and that predicted by Nicholas, especially at low values of bearing eccentricity. The reason for this difference could be that Nicholas considers there to be

damping in the oil film of all the pads, whereas the authors are unable to quantify damping for a particular pad if it does not generate any hydrodynamic pressure and so will not force the pad against its housing. Under such conditions, unloaded pad radial positions are then unknown and, although there may be some associated damping, the authors were unable to quantify it and assigned a zero value for those particular pads.

Dr. Nicholas suggested that the authors compare dynamic characteristics for the variable viscosity case against the constant viscosity case. For this, a study has been carried out on a 5-inch tilting-pad bearing referred to in Ref. (B2).

The bearing details are:

Bearing diameter = 5 in

Bearing length = 5 in

5 pads—centrally pivoted

Pad arc 60° —loaded on pad

$$\text{Pad preload} \left(m = 1 - \frac{c'_r}{c_r} \right) = 0$$

Load = 3420 lbf

*As the comparison is made in dimensionless terms, the authors in using such terms had to "force" the viscosity to be the same value on all pads (unlike the approach in the authors' paper).

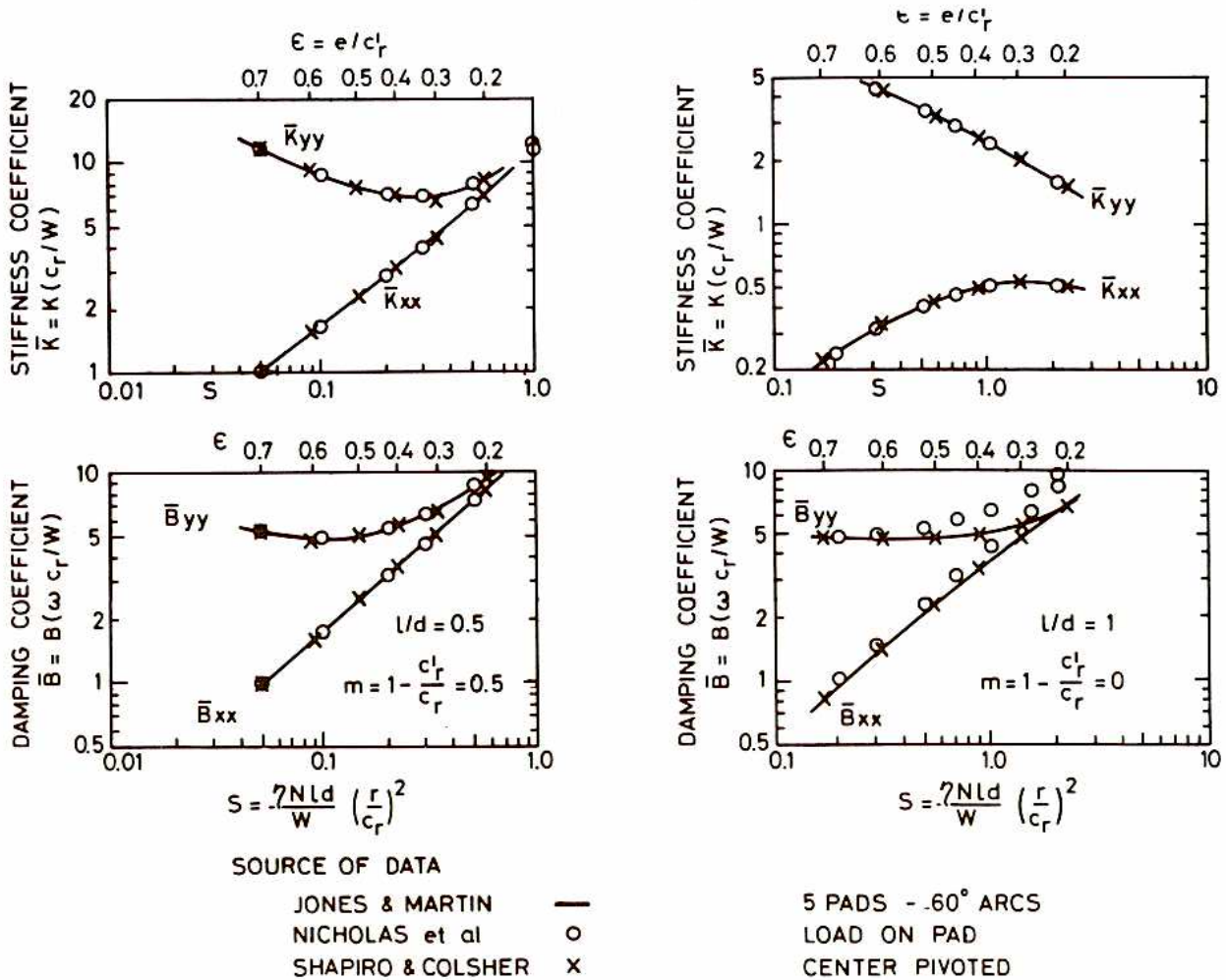


Fig. B2—Comparison of predictions for dynamic performance characteristics of tilting-pad journal bearings.

TABLE B1 COMPARISON OF STIFFNESS AND DAMPING IN CONSTANT AND VARIABLE VISCOSITY CASES.			
	CONSTANT VISCOSITY CASE (SAME ON ALL PADS)		VARIABLE VISCOSITY CASE (DIFFERENT ON EACH PAD)
	SHAPIRO AND COLSHER	AUTHOR	AUTHOR
Load (lbf)	3433	3420	3420
K_{xx} (lbf/in)	2.28×10^5	2.22×10^5	3.05×10^5
K_{yy} (lbf/in)	3.01×10^5	2.99×10^5	2.93×10^5
B_{xx} (lbf.s/in)	1896	1861	2545
B_{yy} (lbf.s/in)	6172	6141	6057
Effective Viscosity (centipoises)	13.8	13.8	13.4/18.5/22.3/22.1/19.3

Speed = 5000 rev/min

Radial clearances $c'_r = c_r = 0.005$ in

For the constant viscosity case in Ref. (B2), a viscosity value of 2×10^{-6} lbf-sec/in² (13.8 centipoises) was used and the resulting bearing eccentricity ratio was 0.6. In order to select an oil, the authors chose a grade to give an eccentricity ratio of 0.6 when using their variable viscosity (from pad to pad) procedure. The oil grade required to satisfy these conditions was rather thick, an ISO VG 100 oil (100 centistokes at 40°C and 11 centistokes at 100°C). The results of this study are shown in Table B1.

From this table, it can be seen that the authors dynamic characteristics, for the constant viscosity case, compare well with those of Shapiro and Colsher. In using variable viscosity, for the particular bearing considered, a higher stiffness and damping results for the xx values (about 40 percent above that for the constant viscosity case). There is very little difference in the yy values, however.